A Logical Proof of the Existence of God

By

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I. The Historical Context.

There is a continuous history of proofs of the existence of God starting with Aristotle's well known proof of the existence of an uncaused cause (his *prime mover*).

Aristotle’s proof is based on an infinite regression principle and uses ***attributional logic.***

Attributional logic is the logic that deals exclusively with ***properties*** of objects. Example:
"------ is green" attributes or assigns the property of greenness to any object whose name is substituted for the blank.

With the appearance of *Najat* (salvation) by the great Muslim philosopher Avicenna (980-1037) comes the first use of ***relational logic*** as a basis of a proof of God’s existence. Avicenna thereby avoids any appeal to Aristotle’s infinite regression principle.

Relational logic includes attributional logic but goes beyond the latter by treating also relations or links between two existents. Example:

“----- is a brother of _____”

Avicenna’s proof was a small part of an ambitious philosophical program of reconciling revelation (i.e., the Koran) with science (essentially Greek philosophy, especially Aristotle).

The immediate successor of Avicenna was the Arabic-speaking Jewish Rabbi Maimonides (1134-1204). In his work *Guide to the Perplexed,* M. presents a reformulation of Avicenna’s proof but reverts to attributional logic and an appeal to Aristotle’s infinite regression principle.

Taking Avicenna’s work as a model, Maimonides conceived of his own program of reconciling the Torah with Aristotelian philosophy.

The famed Catholic philosopher and theologian Thomas Aquinas (1225-1274) followed on the heels of Maimonides, and Thomas' *Summa Theologicae* represents the latter's attempt to reconcile Greek philosophy with the New Testament of Christianity.

The *Summa* contains three ‘ways’ of knowing (proving) God. Thomas’ ‘third way’ is his formulation of the Avicenna proof and, like M.’s, reverts to attributional logic and appeal to the principle of infinite regression.

The treatment of God's existence by later philosophers such as Descartes, Leibniz, and Kant used attributional logic and appealed to the infinite regression principle, as well as relying on ***modal logic***.

The logic of modalities involves such notions as necessary existence or contingent existence, instead of simply existence or non-existence. These modal notions are so vague that there is, even today, no universally agreed upon system of modal logic.

Thus, none of Avicenna’s successors used or understood his method. Even Avicenna did not see his method as participating in a new logic but only as a novel way he had found to treat the specific question of God’s existence.

II. The Modern Period: the advent of relational logic.

The first systematic treatment of relational logic was in *Begriffschrift* (1879), by G. Frege. Begriffschrift means "concept writing".

Frege's basic idea was that written language was twice removed from its content, being a transcription of the phonemes of speech, which in turn, represent ideas. Frege originated the notion of a ***formal language*** in which each symbol represents exactly one logical idea.

Such formal languages now constitute both the theoretical foundations (architecture) and the practical foundations (programming languages) of computer science.

The successors to Frege were B. Russell, E. Zermelo, and finally J. von Neumann in his doctoral thesis in 1925, which, in the opinion of many, carried relational logic to its most refined form.

It was under von Neumann that the first electronic computer, the Eniac, was conceived and built at Princeton (1938-1947). This and all subsequent computers are based on relational logic and could not exist had relational logic not been conceived.

III. The Power of Relational Logic.

There are several ways of assessing and understanding the increased power of relational logic over attributional logic.

AL is decidable: there exists a computer algorithm ***A( )*** such that, given any statement in the language of AL, the algorithm will terminate in a finite time and yield 1 if the statement is a truth of AL, and 0 if it is not.

RL is semidecidable. This means that there exists a computer algorithm ***S( )*** with the following property: if it terminates when applied to any statement in the language of relational logic, then that statement is a truth of relational logic. In case of nontermination we can draw no conclusion.

Furthermore, it is known (and proved by Church in 1936) that there does not exist and cannot exist a decision algorithm for RL. RL is thus essentially undecidable.

More importantly for philosophy, AL and RL lead us to ask quite different kinds of questions of reality. In AL, we seek to know an object by asking what are its intrinsic properties. In RL, we want to know how the object relates to other objects.

It turns out that the relational approach often yields more useful information while avoiding such metaphysical clichés as "fire burns because it is the nature of fire to burn."

But why logic?

Because logical deduction allows us to derive the unobvious from the obvious through a series of individually obvious steps.

IV. The Proof Itself, part 1: the causality relation

Our proof depends on exactly four explicit principles, one extralogical principle and three logical principles. As we present each principle, we will see that it is empirically grounded.

We say that a metaphysical principle is empirically grounded if the restriction of the principle to physical reality yields a known truth of empirical science. It them becomes a metaphysical generalization of an empirical law.

This way of doing metaphysics is part of a general philosophical method, called *Minimalism.* Our articulation of our logical proof of God’s existence is, in general, an illustration of Minimalism at work.

We begin with our one extralogical assumption:

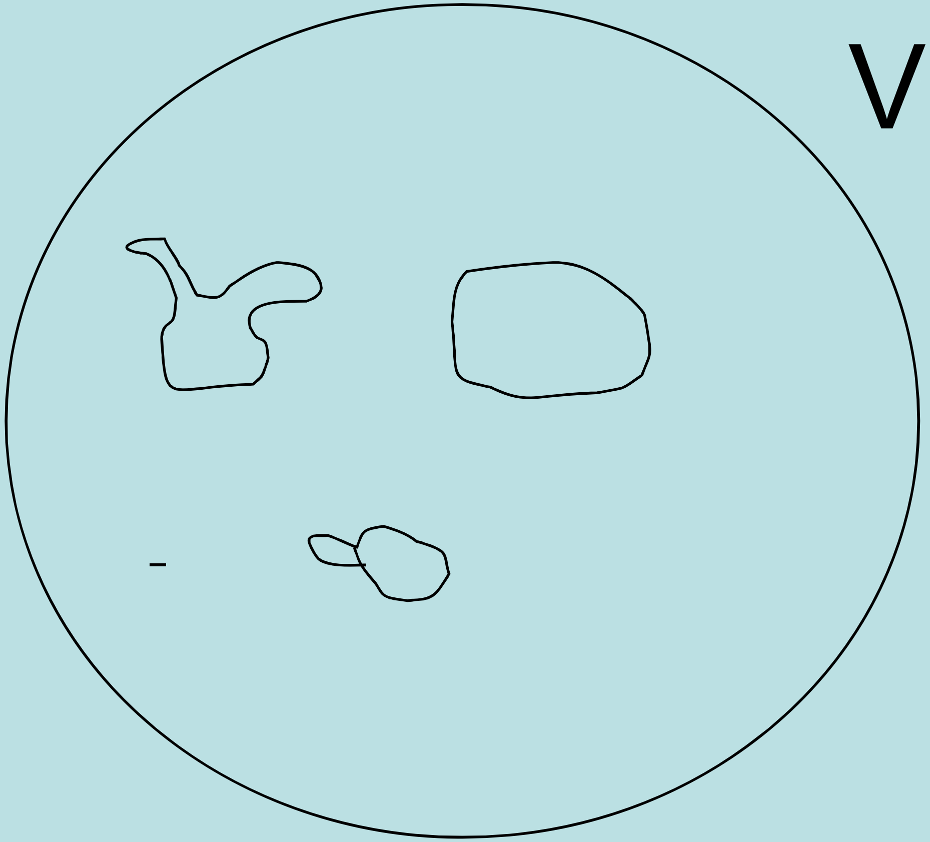
P.0. Something exists (there is not nothing).

P.0 is obviously empirically grounded and is, in fact, obviously true. However, the spirit of Minimalism is that we make all assumptions explicit, regardless of degree of obviousness. Our assumption of the extralogical P.0 makes our proof a *cosmological* proof rather than an *ontological* proof.

We now define *reality* as the totality of actual existence = everything there is (or was or will be).

A *phenomenon* is some nonempty portion of reality.

Example: Let V symbolize reality, the latter being conceived as the interior of the larger circle.



V

Every subdomain of V of any
shape form or fashion is a phenomenon.

We now consider a binary relationship $\rightarrow$ called *causality* which may hold between any two phenomena A and B. If the relationship $A \rightarrow B$ does indeed hold, then we say that A causes B. This means “B exists by virtue of A.”

Generically, causality is a logical relation, but this relation has an empirical counterpart in the physical world: If A$\rightarrow$B holds, then it can never occur that A holds without B holding.

“Never A without B” is thus a necessary (but not sufficient) condition for A$\rightarrow$B to hold. This is like semidecidability. If ever we observe an instance of A without B, then we know certainly that A does not cause B. But in the absence of such a clear counterexample, we can draw no conclusion either way.

However, the empirical requirement that “never A without B” is clearly enough to ground empirically the causality relationship. The point is that causal links are inferred (logically) and not observed, as Hume already indicated.

Causality is thus a legitimate principle of minimalistic metaphysics. We now proceed with certain definitions related to the causality relationship.

D.0. A phenomenon B is *without a cause* if, for no A, does $A \rightarrow B$ hold.

D.1. B is *caused (other-caused)* if for some $A \neq B$, $A \rightarrow B$ holds and $B \not\rightarrow B$ does not hold (i.e., $B \rightarrow B$).

D.2. B is *uncaused (self-caused)* if $B \rightarrow B$ and never $A \rightarrow B$ for $A \neq B$.

We can now articulate the first of our three logical principles, the *principle of sufficient reason.*

P.1. (POSR) Every phenomenon B is either caused or uncaused (and never both).

P.1 implies that no phenomenon B can exist without a cause, be that cause either wholly within B or (partly or wholly) outside of B. In other words, the situation described by D.0 cannot occur. Either D.1 or D.2 must occur, for any given phenomenon B

P.1 says that the "why" question is always meaningful (even if we never find the answer). If we ask "why B?" the answer "there is no reason, that's just the way it is" is not acceptable. POSR is thus the fundament and basis of (scientific) rationality. It is the essential logical precondition for all of science.

Example: why is grass green? Answer: Because white light contains the full spectrum of all the colors and because the structure of grass contains a substance (pigment) which absorbs all except the green portion of the light spectrum, and reflects the rest (the green portion).

Note: This is a case of other-causation, because self-causation implies that the whole cause be within the phenomenon itself.

Part V. A Last Look at Aristotle.

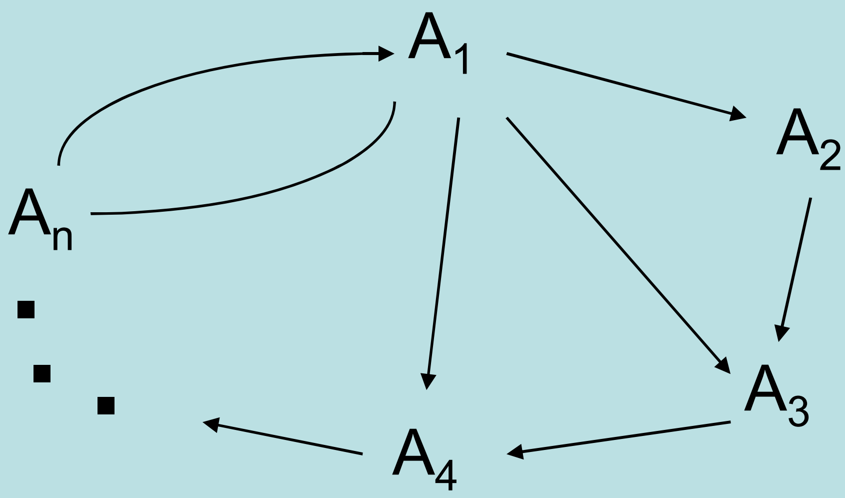
We are now in a position to give a modern version of Aristotle’s first-cause proof in relational logic. This is useful not only for understanding our proof but also as a good exercise in the application of relational logic.

POSR was used in Aristotle’s proof, but without being explicitly identified, only tacitly assumed. The principle is clearly named and identified by Leibniz. Another tacit principle needed for Aristotle’s proof is Transitivity. We posit this as a temporary principle (we will not need it for our proof).

T.1. (Transitivity) If $A \rightarrow B$ and $B \rightarrow C$, then also $A \rightarrow C$.
P.1 and T.1 are all that is needed to prove the following Lemma in preparation for Aristotle’s proof.

Lemma 1. There cannot be any circular causal chain among distinct phenomena.

Proof. Let a circular causal chain $A_1 \to A_2 \to . . . \to A_n \to A_1$ of length n be given. We claim that necessarily $A_1 = A_2 = . . . = A_n$. Indeed, by repeated use of transitivity we have the following relationships.



Thus, $A_1 \rightarrow A_n \rightarrow A_1$. Invoking transitivity yet again, we have $A_1 \rightarrow A_1$. Thus, by P.1, A_1 is uncaused and thus cannot be other-caused. But $A_n \rightarrow A_1$. Thus, A_n is not “other”, i.e., $A_n = A_1$. Indeed, by transitivity, every $A_i \rightarrow A_1$ and thus every $A_i = A_1$ as claimed.

We now posit the second temporary principle, Aristotle's principle of infinite regression.

T.2. An infinite regression of causes is impossible. Precisely, we cannot have an infinitely descending causal chain

$\ldots A_n \rightarrow \ldots \rightarrow A_2 \rightarrow A_1$ where the A_i are all different.

We now use the principles P.0, P.1, T.1, our Lemma, and T.2 to prove Aristotle's theorem:

AT. There exists at least one uncaused (self-caused) phenomenon.

Proof. The proof is by contradiction. Suppose that there is no uncaused phenomenon. By

P.0 we know that at least one phenomenon A_1 exists. By hypothesis, A_1 is not uncaused and thus, by P.1, it is other-caused by some $A_2 \neq A_1$, $A_2 \rightarrow A_1$. But A_2 is also not uncaused (hyp) and thus other-caused (by P.1) by some $A_3 \neq A_2$. Thus, $A_3 \rightarrow A_2 \rightarrow A_1$

Since $A_2 \neq A_1$, it must also be that $A_3 \neq A_1$, because otherwise we would have a circular causal chain among the distinct entities A_1 and A_2. More generally, if we have a causal chain of length n among distinct phenomena $A_n \rightarrow . . . A_3 \rightarrow A_2 \rightarrow A_1$, then our

Lemma tells us that we can add an $A_{n+1} \rightarrow A_n$ where A_{n+1} is different not only from A_n, but from all the other A_i, because if A_{n+1} is equal to any other A_i, then we will have a circular causal chain among distinct phenomena. Thus, if there is no uncaused phenomenon (i.e., if all phenomena are other-caused), then we can construct an infinitely descending causal chain of distinct phenomena, contrary to T.2. Since we assume T.2 true, we conclude that there must be at least one uncaused (self-sufficient) phenomenon.

Evaluating Aristotle’s Proof

1. Burden of proof rests on T.2.
2. A. held that an infinite regress was logically impossible. Modern mathematics shows that this is false. Counterexample, the negative integers:

$\ldots -n < \ldots -3 < -2 < -1 < 0.$

3. However A. only needs the weaker principle that excludes

an infinite regress of *causes.* In this weaker form, the principle is defendable but still controversial.

4. AT does not deny the possibility of many different uncaused causes--even an infinity of them. Contradicts monotheism. Each u.c. is equally a candidate for

Godhood. Thus, if we are consistent monotheists, none of them is a candidate for God (lack of uniqueness).

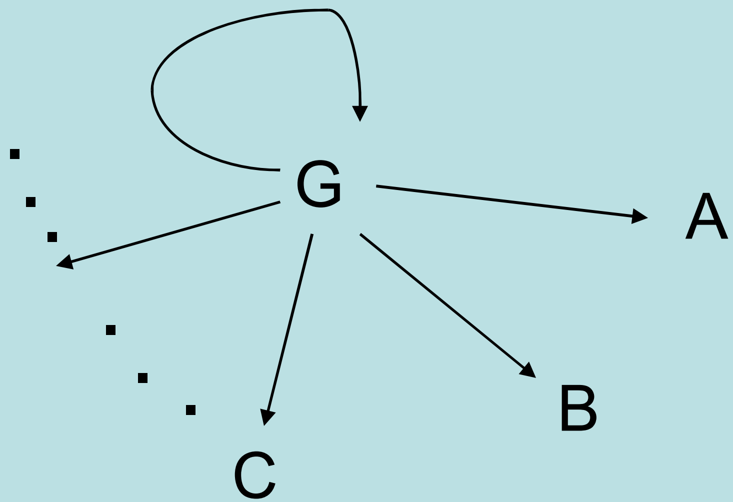
5. Also, AT does not prove that any u.c. is in fact a *universal cause* and thus a candidate for Creator of all things.

6. Thus (Avicenna’s criticism), ***even if we grant the cogency of AT, this theorem does not really do the job of proving that God exists because the phenomenon whose existence is proved (at least one u.c.) does not satisfy the minimal criteria for Godhood.***

VI. Completing Our Proof.

Avicenna’s criticism of Aristotle’s proof raises the question: How will we know that we have proved the existence of God? We must now give a precise logical definition of God so that we will know when and if we are successful.

We will shortly give such a definition, but for the moment the minimal conditions are that God must be a phenomenon G that is unique, uncaused, and a universal cause, i.e., the ultimate cause of all phenomena in existence. The following diagram illustrates these relationships:


G
A
B
C

Note: It is important to realize that causality need not be *direct.* Suppose that $A \rightarrow B$, $A \neq B$, and that there is some C different from A and from B and such that $A \rightarrow C \rightarrow B$. We say that C is an interpolant cause between A and B and that the causality between A and B is indirect.

If A$\rightarrow$B and there is no interpolant cause C between them, then we say that the causality of B by A is direct. Thus, to say that G is a universal cause does not mean that God has directly caused every phenomenon.

It means that every existing phenomenon is the end effect of a causal chain, of possibly infinite length, starting with G. Many useless philosophical controversies result from a failure to understand the distinction between direct and indirect causation.

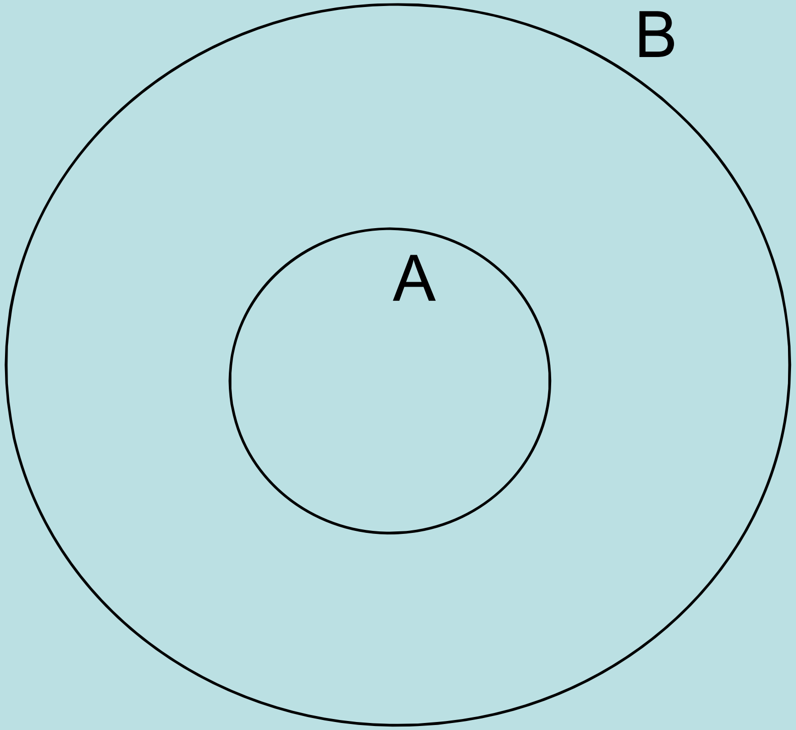
Following Avicenna, we now introduce a second binary relationship $\in$ which may hold between two phenomena A and B. If $A \in B$ holds we say that "A is a component of B". Given B, if $A \in B$ for at least one A, then we say that B is *composite.* Otherwise, B is *simple (noncomposite).*

Composites are phenomena which have parts. All known physical phenomena are composites except, possibly, the elementary particles of quantum mechanics (e.g., quarks or photons). The question of the simplicity of these latter particles is still controversial.

A composite phenomenon will also be called a *system.* We use the componenthood relationship $\in$ to define another relation that holds only between systems.

D.3. When every component $E \in A$ is also a component $E \in B$, we write $A \subset B$ and say that A is a *subsystem* of B.

The following diagram illustrates the difference between componenthood and subness.



The components of B and A are the points inside their respective boundaries. Every point in A is certainly in B so that $A \subset B$ clearly holds. But $A \notin B$ since the components of B are points, and A is a circle, not a point.

However, we do use the word "part" to refer indifferently to components or to subsystems.

D.4. If either $A \in B$ or $A \subset B$ holds, then we say that A is a *part* of B.

Note: in spite of the distinction between component and subsystem, it is nevertheless possible for something to be

both a component and a subsystem.
Example: The digestive system is both a component of the body as an organism and also a sub-system since the components (organs) of the digestive system are also components of the body.

With this terminology established, we can now state our last two logical principles. For ease of reference, we also restate P.1.

P.1. (POSR) Every phenomenon A is either uncaused or caused, and never both.

P.2. (Potency) If $A \to B$ and if either $E \in B$ or $E \subset B$ (i.e., E is a part of B), then $A \to E$.

P.3. (Limitation) If $E \in B$, then $B \not\to E$.

Comments on definitions:

1. P.2 asserts that our causality relation is *complete causality.* In science, causality corresponds to Aristotle's notion of efficient cause. The efficient cause is the straw that finally breaks the camel's back. The complete cause is all of the other straws which, together with the last one, have broken the back of the camel.

We can thus make the following equation: IP + EC = CC,$\rightarrow$RP. “The initial phenomenon plus the efficient cause equals the complete cause, which causes the resulting phenomenon, RP.” In science, IP is assumed already given and we are trying to determine EC in order to obtain CC and thus RP.

2. P.3 asserts that a whole B cannot be the cause of one of its own components E. This is because the whole does not even exist (to be a cause of anything) until all of its components exist.

3. Notice that nothing excludes that a component may be the cause of a whole of which it is a part.

Clearly, both P.2 and P.3 are empirically grounded. P.2 is virtually a definition of the notion of complete cause and P.3 is, essentially, a special case of the second law of thermo-dynamics, which negates the possibility of purely “holistic” causality, i.e., the transfer of order from a whole to a proper part, without any input of organizing energy from outside the system.

Finally, we have:

D.5. By God, symbolized G, we mean a unique, self-caused (uncaused), noncomposite, universal cause, if such a phenomenon exists.

We will prove by pure, formal nonmodal logic that P.0&P.1& &P.2&P.3 imply that G exists. Let us recall our observation that logic gets the unobvious from the obvious. P.0-P.3 are so obvious that most people would not even feel the necessity to assume them explicitly.

Yet, the conclusion that G exists is far from obvious. This illustrates the power logic has when it is properly deployed. Before giving the proof, we need one further definition, which will enable us to give a more formal, precise definition of the universe V of existence.

D.6. A phenomenon B is an *entity* if it is a component of at least one other system A: for some $A \neq B$, $B \in A$. Thus, all components are entities and all entities are components. We assume that all non-composites are entities. We thus have the following tripartite ontology.

For noncomposites B, $? \notin B \in A$ for some $A \neq B$. (noncomposites are always entities). For composite entities B, $E \in B \in A$ for some $E \neq B$ and for some $A \neq B$. For non-entity composites B, $E \in B \notin ?$ for some $E \neq B$. We now (re)define the global phenomenon V in these terms.

D.7. Let V be the phenomenon whose components are precisely the (all) entities. Now, to be a component is to be an entity and to be an entity is to be a component of V. Thus, to be a component (of something--anything) is to be a component of V. We thus have:

Lemma 2. Every phenomenon B is a part of V.

Proof. If B is an entity (composite or not), then $B \in V$ by definition. If B is composite (whether an entity or not), then every component $E \in B$ is an entity (def.) and thus a component of V (def.). Hence (def.) $B \subset V$. In either case, B is a part of V.

This terminology allows us to restate P.0 in a more formal and elegant manner:

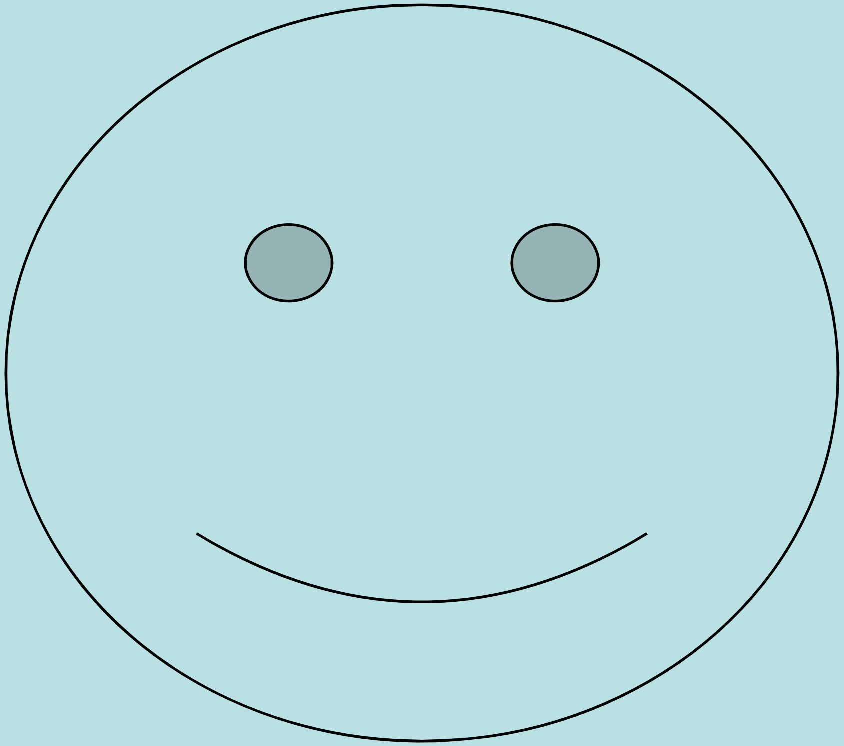
P.0. V is composite.

We now state:

Theorem 1. Assuming P.0-P.3 and our various definitions, then there exists a unique, non-composite, universal cause G.

Proof. By P.1, V is either self-caused or other-caused. Suppose $V \to V$. By P.0, V is composite. Thus, $E \in V$ for some E. But then, by P.2, $V \to E \in V$, which contra-dicts P.3. Thus, $V \to V$. Hence, by P.1, $G \to V$ for some pheno-menon $G \neq V$. Like every pheno-menon, G is a part of V. Thus, by P.2, $G \rightarrow G$. G is therefore self-caused. But this means that G is noncomposite, since $E \in G \rightarrow G$ for some E implies, by P.2, that $G \rightarrow E$, contradicting P.3 ($G \not\rightarrow E \in G$). G is also universal, because every phenomenon B is a part of V (Lemma 2). Thus, by P.2 and $G \rightarrow V$, it follows that, for every phenomenon B, $G \rightarrow B$. Finally, G is the unique uncaused phenomenon, for suppose that, for some phenomenon G_1, $G_1 \rightarrow G_1$. Now we have already estab-lished that G is universal. Thus $G \rightarrow G_1$. By P.1, G_1 cannot be both self-caused and other-caused. But G is a cause of G_1. Thus, G is not “other”, i.e., $G=G_1$ as claimed.

We have thus proved the existence of a unique uncaused, noncomposite, universal cause.



VII. Comments and Evaluation

The strength of the proof

1. Avicenna actually used a complicated system of modal-ities, which we have eliminated entirely. He also made many further assumptions, which we have shown either to be unnecessary or else deducible from our assumptions.

2. To facilitate discussion, we henceforth assume P.0 to be given and true (something exists). Our proof thus shows that (P.1&P.2&P.3)$\Rightarrow$G. The logical cogency of the proof is beyond question: the proof can be and has been totally formalized.

3. The proof is not an abstract word game. If the three logical principles P.1-P.3 are valid (true), then our G does in fact exist.

4. Anyone who rejects the conclusion G has only one rational option. That person must deny one or more of P.1-P.3:

$(P.1 \& P.2 \& P.3) \Rightarrow G$, thus, $G \Rightarrow$ P.1 or P.2 or P.3. But to deny a proposition P is to affirm that its negation $\neg P$ is true. Thus $\neg G \Rightarrow$ $(\neg P.1)$ or $(\neg P.2)$ or $(\neg P.3)$. This is not such a simple affair as it might seem at first.

5. Indeed, each of the P.i is a universal statement, i.e., a

statement that makes no existence assertion. The negation of such a universal statement is always an existence statement. Thus, to deny any of the P.i is to commit oneself to the existence of certain abstract entities. For example, if I deny P.3, then I must believe that somewhere in Plato’s universe of forms there is a system B which is the cause of one of its own components. Certainly no physical system that we have ever observed or postulated has such a property, but if I insist on negating P.3, I must believe that such a thing really exists.

Or, if I negate P.1, I am committed to believing that there is some phenomenon B which exists without any cause or reason whatever. Such an exception to the POSR would, itself, but a good candidate for God, because according to the principles of modern science, it could not be any physical system. Indeed, God as we have defined Him is a much more reasonable hypothesis than is such a B. A similar remark holds for P.2.

6. Let us sum up. Each of the P.i is empirically grounded and thus far more reasonable than its negation. Moreover, the negation of any P.i commits us to belief in an abstract entity satisfying highly unlikely conditions.

Since the conjunction of the P.i also imply the existence of the abstract entity G, we conclude that "nihilistic atheism," i.e., the refusal to countenance the existence of any abstract, non-observable entity, contradicts pure logic itself -- independently of any assumptions whatsoever.

In other words, strict materialism is logically untenable. Atheism involves existential commitment and cannot be consistently maintained as the denial of belief in any nonobservables. At the very least, our argument definitively shifts the existential burden of proof from the theist (who accepts the P.i and thus the existence of G) to the atheist, who must now justify his irrational preference for believing in one of the bizarre phenomena posited by one or more of ¬P.i.